Robust Content-Dependent Photometric Projector Compensation

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Abstract

We wish to compensate for irregularities in the output of digital projectors that occur when they are used in non-ideal situations, such as those with varying surface reflectance and ambient light. We transform the image to be displayed into a compensation image that will produce the desired appearance. In contrast to previous methods, the transformation is based on both a radiometric model of the system and the content of the image. We present a five-stage framework for performing content-dependent photometric compensation, and the details of a specific implementation. The original image is converted to a perceptually-uniform space, the desired chrominance is fitted to the gamut of the projector, a luminance range is calculated within which the fitted chrominance values can be produced, the original luminance is fitted to that range, and finally the fitted values are converted to a compensation image. Our method balances strict compensation against dynamic range in the final output, so we can produce a good result even when removal of all visible spatial variation is not possible.

1. Introduction

Projectors are becoming smaller and cheaper, and therefore more popular. This will cause them to be used in more situations that deviate from the ideal case of a perfect white screen and low ambient light [12, 1], especially when the projectors are steerable [7], or portable [10]. In these situations we would like to compensate for the resulting irregularities in projector output.

To project corrected images onto arbitrary surfaces geometric and photometric compensation are necessary. Here we will concentrate on photometric compensation, whose aim is to produce a compensation image that will produce the desired result when provided as input to the projector. There are two main pieces of data that are used to calculate the compensation image: the radiometric model that defines the range of colours that the projector can actually produce, and the image which defines which colours the viewer should see.

Almost all previous photometric calibration methods have been content-independent, with a single transformation being calculated from the radiometric model during an off-line stage then applied to all input images. Here we describe a compensation method that also considers the content of the image. The tradeoff is that a content-dependent method will require more computation during the on-line stage, but the compensation is tailored to the current image so in general a better result should be possible.

The contributions we present here are a framework for performing robust content-dependent photometric projector compensation, and a specific implementation which we demonstrate with simulated and real results. Our framework consists of five stages that take the original image and result in the compensation image. In particular we transform the chrominance and luminance values separately in two stages that fit the original values to the range of outputs that the projector can produce at each display pixel.

If the projection system is close to ideal it will be pos-
sible to completely remove any visible irregularities in the final output. In more difficult situations, for example where the surface contains very saturated colours, it will not be possible to produce the same result as would occur with a perfect white surface. In this case we wish to balance the various constraints that affect the quality of the final result, to allow image quality to degrade gracefully as the conditions become adverse for projection.

After mentioning previous work we describe some background to our system, including the off-line calibration steps that are used to characterize the response of the system to the projector inputs. Then we describe our compensation framework, a specific implementation of it, and results from that implementation, before finishing with conclusions and ideas for future work.

2. Previous Work

A digital projector requires geometric and photometric calibration to produce a correct image on an arbitrary surface. These stages respectively ensure that a pixel appears in the desired position, and has the desired colour. Much work has been completed on geometric calibration for stationary [11] and moving projectors [10], so we will not cover that aspect here. We will focus on the photometric aspect.

If a projection system is spatially uniform a single linear transformation can be used to represent the relationship between projector input and colour on the surface [14, 16]. Tsukada and Tajima [15] use a similar model to perform compensation for a uniform coloured surface.

A linear transformation can be calculated for every pixel of a projected display to perform compensation in the presence of spatial variation [3, 2, 1]. Previous systems that have used such a spatially varying linear radiometric model of the projection system have applied a spatially uniform transformation to the desired image to fit it to the projector range. This transformation can be chosen manually, or based on a simple calculation such as interpolating the desired luminance between two values which represent a luminance range that is possible at all points on the display. In this case choosing too small a range of outputs will waste the dynamic range of the projector, and choosing too large a range will cause clipping at some points.

Properties of human perception have been used to relax the need for a strictly uniform image transformation: Majumder and Stevens use a spatially varying luminance transformation [6] which is subject to a gradient constraint.

The only content-dependent system we know of is that of Wang et al. [17] who use a spatially varying error threshold based on a model of the human visual system to process the image content. They then allow clipping of the final pixel values to occur in areas where the viewer will not notice. However, they use a restricted radiometric model and a spatially uniform image transformation.

Our system uses a spatially varying radiometric model of the projection system, and a spatially varying image transformation, to perform content-dependent compensation. It also allows physical limits imposed by the output range of the projector, and perceptual limits due to the tolerance of a human viewer to variation in luminance and chrominance, to be balanced against the goal of maximizing the dynamic range of the final result.

3. Background

The image to be displayed by the projector will be supplied in RGB format, and the projector takes RGB as input, but these two RGB spaces are device-dependent, and perceptually will be highly non-uniform: a movement of the same distance will correspond to very different perceptual changes for different starting points and directions. We therefore transform to a perceptually-uniform space where a simple error function can be applied at any point.

We define the relationship between projector inputs and the colour on the surface using a radiometric model, and calibrate this in an off-line stage by observing the projector outputs with a camera. We assume that the environment and image are static, the projector has three primary colours, and its light combines additively on the surface which is Lambertian.

3.1. Colour Spaces

Humans have trichromatic vision, so colour spaces are defined with three dimensions. The projector accepts RGB inputs: mixtures of red, green, and blue primaries. We define each channel to be in the range [0,1], so the range of possible inputs is the unit cube (Figure 2(a)).

RGB is device-dependent, but we want to measure the colour on the screen in a device-independent manner, so we use CIEXYZ [18]. The conversion from RGB to CIEXYZ is a linear one, as described in the next section, so the cube in RGB space transforms to a parallelepiped in CIEXYZ space (Figure 2(b)). This shape represents the range of possible outputs that the projector can produce: the gamut. CIEXYZ space is not perceptually uniform, so to evaluate the perceptual effects of an image transformation we can convert to a space such as CIELUV [18]. This colour space was designed so that an equal movement in any direction corresponds to an equal perceptual difference for the viewer, but the transformation from CIEXYZ is not a simple linear one so the resulting gamut has a more complex shape (Figure 2(c)). As is typical for such perceptually-based spaces, CIELUV defines one dimension for perceived luminance, and two for chrominance.
3.2. Radiometric Model

The output of a projector is generally not a linear function of its input, so we first measure the response functions for the three channels, red, green, and blue. The projector’s brightness will vary across the display, but the shape of the response functions has been shown to be spatially uniform [5]. We apply the inverse response functions before the final RGB compensation image is sent to the projector. We will subsequently assume a linear relationship.

The relationship between projector input and output is different for every pixel because of spatial variation in surface reflectance, ambient light, and projector brightness. The input to the projector is an RGB pixel. We assume the projector has three primary colours, so the output in CIEXYZ space can be related to the input using a linear transformation [14]:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = M \begin{bmatrix} R \\ G \\ B \\ 1 \end{bmatrix}. \quad (1)$$

Here $M$ is a $3 \times 4$ matrix. Grossberg et al. [3] justify the use of this type of transformation for calibrating a projector. Our characterization of the projection system this way is the same as theirs; the difference is the way we fit the image to the range of possible outputs. Rather than manually selecting a uniform range for the output image, we use an algorithm that considers the content of the image and the projector limitations to produce a spatially-varying mapping between input and output images.

3.3. Calibration

Initially we obtain the projector response functions for the three channels by measuring the output of a point on the screen with a colorimeter. A digital camera and high dynamic range imaging techniques could also be used [9].

We obtain the 12 elements of $M$ in Equation 1 for each pixel by displaying several uniform images from the projector and capturing them with a camera calibrated to measure CIEXYZ colours. The projector inputs and measured colours are known, so we can obtain a solution to the linear system in Equation 1. Our calibration method differs from the previously published one [3] in that we use high dynamic range imaging [13] to capture the projector outputs over a range of luminance levels, and we use more than four uniform images to obtain a least-squares solution to Equation 1.

4. Compensation Framework

We define and calibrate a radiometric model of the relationship between projector inputs and resulting colour on the display surface as described the previous section. When a new image is received we must then use it, and the calibration data, to generate a compensation image. The input to this process will be an RGB original image, and the output will be an RGB compensation image.

4.1. Overview

The original image will give rise to a desired appearance that we wish to achieve, and the radiometric model defines a space of possible appearances that the projector can produce. We will transform the desired appearance to a point within that space, subject to various constraints. For an ideal projection configuration the transformation can simply be the identity since the projector can produce all desired appearances, but for realistic configurations significant tradeoffs may have to be made between maintaining fidelity to the original image and hiding irregularities such as variation in surface reflectance and ambient light.

Our method starts by converting the original RGB image to a desired appearance in a device-independent colour space. We then take the approach, popular in image coding and compression, of splitting the original image into luminance and chrominance channels. These two are treated separately as the original image is transformed to fit the range of output that projector can produce. We consider the luminance information to be more salient so we put more effort into maximizing its range. The process ends by converting the image to projector RGB to generate a compensation image. We have five stages in the compensation process:

- **Desired appearance** The original RGB image is converted to a device-independent colour space to specify the desired appearance under ideal conditions.
• **Chrominance fitting** A transformation to the chrominance values is chosen to fit the desired values to the projector’s spatially varying gamut.

• **Range calculation** The fitted chrominance values and projector gamut define a minimum and maximum luminance at each pixel.

• **Luminance fitting** The original luminance channel is fitted within the calculated range.

• **Compensation image** The transformed image is converted to an RGB compensation image.

There is a tradeoff between maximizing the ranges of chrominance and luminance. By choosing a smaller range of values in the chrominance fitting stage we will subsequently be able to achieve a higher luminance range. Also, some projection situations will be close to the ideal so we can fully compensate for any spatial variations, but others will be more difficult, for example when the projection surface has a vivid pattern. A key aim of our method is that it produces a good result in favourable situations, and degrades gracefully for situations that are more difficult. The next five subsections describe our compensation framework in more detail.

### 4.2. Desired Appearance

The original image is provided in RGB which is device-dependent. We convert this to a colour space which is device-independent, so that the desired appearance is specified precisely, and we choose a space that is perceptually uniform so that error functions can be applied in the later stages. This conversion is equivalent to the first step of various image compression algorithms: an initial lossless conversion to space in which perceptually-motivated transformation and quantization can be applied to the image.

The conversion from RGB to the device-independent space will not consider the calibration data from the projector, so the desired appearance will be the one obtained in the ideal condition with a perfect white screen, no ambient light, and uniform projector output that can produce all desired chrominance values. The following stages will modify this appearance to fit what the projector can actually produce.

### 4.3. Chrominance Fitting

The properties of the projection system will determine its gamut at each pixel (Figure 2). For chrominance fitting we consider the shape of this gamut in two dimensions. The transformation from RGB to device-independent colour takes no account of the gamut of the projector. For instance, using a red projection surface will cause the gamut to be shifted towards the red part of chrominance space, and the size of the gamut will also be affected. If we try to produce chrominance values outside the gamut, clipping will occur resulting in visible artifacts, so we scale and translate the desired values to fit into the gamut obtained during calibration. The chrominance transformation should satisfy four properties:

• **Gamut** Chrominance values far outside the projector gamut will be clipped severely, producing artifacts, so they should be avoided.

• **Uniformity** A small amount of spatial variation in the result will be imperceptible to the viewer, but this variation should be limited.

• **Extent** The range of final chrominance values should be similar to that of the original image.

• **Deviation** The chrominance values after fitting should not be too far from their original positions.

In non-ideal projection situations these four properties must be balanced against each other.

### 4.4. Range Calculation

The minimum luminance that can be produced at a display pixel is determined by the ambient light, and the maximum luminance by the sum of the ambient light and the maximum projector output. These two points are labelled ‘black’ and ‘white’ in Figure 2(c). For chrominance values away from the black and white points the range of luminance is lower. For example, to make a saturated red only the red channel of the projector is used while the green and blue channels are switched off. The achievable luminance range therefore depends on the shape of the gamut and the desired chrominance.

Once the chrominance values for the output have been chosen during the chrominance fitting stage, we use a range calculation stage to determine, independently at each pixel, the minimum and maximum luminance available.

### 4.5. Luminance Fitting

We now have a desired luminance from the original image, and a spatially varying luminance range. The aim of the luminance fitting stage is to fit the former to the latter. The choice of luminance transformation should balance four properties similar to those for the chrominance fitting:

• **Gamut** Luminance values far outside the previously computed range should be avoided.

• **Uniformity** Perceptible spatial variation in the output should be limited.

• **Extent** The dynamic range of the result should be close to that in the ideal condition.

• **Deviation** The difference from the original luminance should be small.
For a projection system with a perfect spatially uniform response this stage will make little difference to the result. However, for a surface with widely varying reflectance it may not be possible to choose a luminance transformation that gives a large dynamic range in the output and also hides the variation in the underlying surface. In this case we wish to balance the perceptible departure from uniformity against the dynamic range of the result.

4.6. Compensation Image

Once the final device-independent pixel colours have been chosen these are converted to an RGB compensation image using the inverse of the the radiometric model (Equation 1). Clipping of fitted colours to the projector’s gamut can either be done in the perceptually uniform space, which can be justified theoretically, or in RGB space, which is computationally cheaper.

5. Implementation

This section describes an implementation of the five-stage compensation method described in Section 4. A simple implementation might make no change to the original chrominance in the chrominance fitting stage, and in the luminance fitting stage simply select a spatially uniform scale and offset. This would be equivalent to automating the choice of luminance range that is implicit in previous methods that have used a spatially varying radiometric model [3].

In the method described below we use a spatially uniform transformation for chrominance fitting, and a spatially varying transformation for luminance fitting. For our device-independent colour space we use CIELUV.

5.1. Desired Appearance

When a new image is received we first convert it from RGB to CIELUV. We assume the image is in sRGB, which provides a mapping from RGB to CIEXYZ by defining a ‘gamma function’ and the CIEXYZ values for the primaries. We use the standard transformation from CIEXYZ to CIE 1976 LUV space except that we define $L$ to be in the range [0,1] rather than [0,100]:

$$ L = 1.16(Y/Y_n)^{1/3} - 0.16, $$
$$ u = 13L^*(u' - u'_n), $$
$$ v = 13L^*(v' - v'_n). $$

We choose the reference luminance $Y_n$ to be the maximum luminance possible over the display, and the white point $u'_n, v'_n$ to be CIE D65. Converting the image in this way results in a desired colour $(L_0, u_0, v_0)$ for every pixel on the display.

5.2. Chrominance Fitting

To define the range of chrominance values the projector can produce at each display pixel we project the three-dimensional CIELUV gamut onto the plane $L = 0$ to obtain a two-dimensional gamut in $uv$ space. We represent this gamut with a set of lines that form the boundary (Figure 3).

During chrominance fitting we compute a spatially uniform chrominance transformation that is applied to the original chrominance $(u_0, v_0)$. It consists of a uniform scale $s$ and a translation $(a, b)$:

$$ \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = s \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}. $$

The lines forming the two-dimensional gamut are computed during calibration and represented in the form $(l_a, l_b, l_c)^T$, normalized so that $l_a^2 + l_b^2 = 1$. Then the signed distance of $(u_0, v_0)$ from the line is $l_a u_0 + l_b v_0 + l_c$. The distance between a transformed chrominance point and a line is $l_a (u_0 s + a) + l_b (v_0 s + b) + l_c$, which can be expressed as $r \cdot m + l_c$, where $r = (l_a u_0 + l_b v_0, l_a, l_b)$ and $m = (s, a, b)^T$.

We define the following error function to judge how good a particular transformation is,

$$ E = c_1 (1-s)^2 + c_2 (a^2 + b^2) + \frac{1}{n} \sum_{\text{pixels lines}} c_3 (r \cdot m + l_c). $$

The first term addresses the extent property: it encourages a large range of chrominance values to be produced. The second addresses deviation: it discourages shifts away from the original white point. The third addresses the gamut: it discourages points outside the gamut being produced. The uniformity property is satisfied trivially because the chrominance transformation is spatially uniform. Variable $n$ is the total number of lines, and $c_1$ and $c_2$ are simple weights. Variable $c_3$ determines how sharply the error increases as chrominance points approach the gamut boundary. The gradient vector and Hessian matrix of $E$ are computed in a pass
over the image using the the formulae
\[ \forall E = 2 \begin{bmatrix} -c_1(1-s) \\ c_2a \\ c_2b \end{bmatrix} + \frac{c_3}{n} \sum r^e c_3(r \cdot m + l_c), \] (5)

\[ H E = 2 \begin{bmatrix} c_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \frac{c_3}{n} \sum rr^e c_3(r \cdot m + l_c). \] (6)

We minimize \( E \) using Newton’s method:
\[ \mathbf{m}_{k+1} = \mathbf{m}_k - \gamma [H E(\mathbf{m}_k)]^{-1} \nabla E(\mathbf{m}_k). \] (7)

We use \( \gamma = 1 \), and \( \mathbf{m}_0 = (1,0,0)^T \). We have found that good values for the constants in Equation 4 are: \( c_1 = 3 \), \( c_2 = 20 \), \( c_3 = 4 \).

5.3. Range Calculation

The limited projector gamut at each pixel means that the projector will be able to produce a chrominance only within a certain range of luminance values. For a particular pixel we will call these low and high gamut-limited values \( G^l \) and \( G^h \). We calculate these in the range calculation stage.

We approximate the gamut in CIELUV space using twelve triangles between the eight corners from the RGB cube (Figure 4). The desired chrominance is tested for inclusion in the six triangles that form the top surface, and the six that form the bottom. If \((u_1, v_1)\) is in a triangle the \( L \) values at the three vertices are linearly interpolated to get \( G^l \) or \( G^h \). If it is not we find the nearest point on the boundary and use the luminance value of that.

5.4. Luminance Fitting

The inputs to the luminance fitting stage are a set of three values for every pixel on the display: the original image luminance \( L_0 \) and the limits \( G^l \) and \( G^h \) computed in the range calculation stage. The original luminance \( L_0 \) will be transformed to the final one \( L_1 \) via a linear interpolation between two spatially varying values \( F^l \) and \( F^h \),
\[ L_1 = F^l + (F^h - F^l)L_0, \] (8)

and the aim of the luminance fitting stage is to find these values (Figure 5). All luminance values are in the normalized range \([0,1]\).

We wish to balance four properties as described in Section 4.5. We introduce an algorithm inspired by the relaxation method used to solve elliptic partial differential equations in shape from shading and optical flow [4].

We define an error function that is a spatial sum of four terms:
\[ e = \sum_i \sum_j (s_{i,j} + d_1 v_{i,j} + d_2 t_{i,j} + d_3 w_{i,j}). \] (9)

Value \( s \) at each point represents the departure from uniformity,
\[ s = \left( (F_x^l)^2 + (F_y^l)^2 \right) + \left( (F_x^h)^2 + (F_y^h)^2 \right), \] (10)
where \( F_x^l, F_y^l, F_x^h, F_y^h \) are partial derivatives. We use the current value of \( L_1 \) based on those of \( F^l \) and \( F^h \) in the definition of the error term \( r \) due to the limited projector gamut,
\[ r(L_1) = \begin{cases} (L_1 - G^l)^2 & \text{if } L_1 < G^l, \\ (L_1 - G^h)^2 & \text{if } L_1 > G^h, \\ 0 & \text{otherwise.} \end{cases} \] (11)

This term is zero when the interpolated luminance is within range, and rises quadratically outside the range. Its derivative is a piecewise linear function. Terms \( t \) and \( w \) cause the deviation of the final luminance from the the original values to be reduced, and the extent—dynamic range—of the result to be increased. We define them as
\[ t = (F^l)^2 + (F^h - 1)^2, \] (12)
\[ w = e^{d_3(F^l - F^h)}. \] (13)
Term $t$ is zero when $F_l = 0$ and $F_h = 1$, and increases quadratically with deviations from those values. Term $w$ prevents the image range from becoming too small by rising sharply as $F_l$ gets close to $F_h$. We take the derivatives of $e$,
\[
\frac{\partial e}{\partial F_l} = 2(F_l - \bar{F}_l) + d_1 \frac{\partial r}{\partial F_l} + d_2 \frac{\partial t}{\partial F_l} + d_3 \frac{\partial w}{\partial F_l},
\]
\[
\frac{\partial e}{\partial F_h} = 2(F_h - \bar{F}_h) + d_1 \frac{\partial r}{\partial F_h} + d_2 \frac{\partial t}{\partial F_h} + d_3 \frac{\partial w}{\partial F_h},
\]
where $\bar{F}_l$ and $\bar{F}_h$ are local averages. Setting these derivatives to zero suggests the following iterative formula,
\[
F_l \leftarrow \bar{F}_l - \frac{d_1}{2} \frac{\partial r}{\partial F_l} - \frac{d_2}{2} \frac{\partial t}{\partial F_l} - \frac{d_3}{2} \frac{\partial w}{\partial F_l},
\]
\[
F_h \leftarrow \bar{F}_h - \frac{d_1}{2} \frac{\partial r}{\partial F_h} - \frac{d_2}{2} \frac{\partial t}{\partial F_h} - \frac{d_3}{2} \frac{\partial w}{\partial F_h}.
\]
We compute the local averages using the filter
\[
\begin{array}{cccc}
1 & 4 & 1 \\
4 & 0 & 4 \\
1 & 4 & 1
\end{array}
\]
and use the resulting average values to compute the partial derivatives on the right of Equation 15. For initial values we use $F_l = 0$, $F_h = 1$, and we find that good values for the constants are $d_1 = 0.01$, $d_2 = 0.0001$, $d_3 = 0.0001$, $d_4 = 6$

5.5. Compensation Image

The final fitted colour at each pixel is $(L_1, u_1, v_1)$. This is converted to RGB before being sent to the projector. We perform gamut clipping by simply clamping output values to the unit cube in RGB space.

6. Results

Figure 1 contains photographs that show our method can compensate for multi-coloured surfaces by removing the effects that would otherwise cause variation in the chrominance of the result. Figure 6 shows that our method can compensate for large variations in the brightness of the projection surface. The projector cannot produce high luminance values on the dark regions of the surface, so the luminance generated at other points must be attenuated to achieve perceptual uniformity.

Figure 7 shows simulated results that demonstrate the tradeoff between dynamic range and strict compensation for spatial variations. In that case, the left side of the surface contains very saturated colours. If a uniform transformation was used on the original image either a large amount of clipping would occur on the left side, or the dynamic range of the result would be very small over the whole image.

The gradient permitted in the final result by the uniformity property is dependent on the size of the image in viewer’s field of view: if the viewer moves closer to the projection surface the spatial frequencies in the image will appear lower. The weighting of uniformity against the other constraints in Equation 9 should therefore be modified by a measurement or assumption of the relative distance between viewer and surface. The chrominance fitting technique described in Section 5.2 requires only a few iterations to converge. The luminance fitting technique in Section 5.4 takes longer with the finite differencing method we have used, but a considerable speed increase should be possible using a multigrid method [8].
7. Conclusion

We have presented a framework for performing photometric compensation for projected images that takes account of the content of the images, and an implementation that uses iterative methods to obtain transformations that fit the chrominance and luminance of the original image into the gamut of values that can be produced by the projector at each display pixel. We have used four goals to guide the fitting methods: the generation of points outside the projector’s gamut should be limited, the result should be uniform enough to hide any spatial variation from the viewer, the extent of the fitted values should be close to that of the original, and deviations from the original values should be small. For an ideal projection configuration it will be possible to meet all of these goals, but in general they must be balanced against one another.

Because our method is content-dependent, spatial processing of the image based on a model of the human visual system [17] could be used to increase the tolerances in luminance or chrominance. Also, it could used with a multi-projector display, in which case the model of the gamut used in chrominance fitting and range calculation (Figures 3 and 4) would have complexity proportional to the maximum number of overlapping projectors.

Our method uses two pieces of data to calculate the compensation image: the radiometric model and the original image. We have assumed both of these are spatially varying but temporally constant. Future work should consider allowing both of these to vary with time. Fujii et al. [2] handle surface reflectance that varies with time for a moving projector by placing the projector and camera coaxially and comparing the predicted display appearance with the appearance captured by the camera. An image that varies with time—video—will require the spatial uniformity constraints we have used here to be joined by temporal ones and the fitting to be performed over three dimensions rather than two.

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References


