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AN INVERSE METHOD TO ESTIMATE THE ACOUSTIC IMPEDANCE ON THE SURFACES OF COMPLEX-SHAPED INTERIORS

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Abstract

Prediction of acoustic response and modal analysis in the interior of rooms are often performed using numerical simulations which require the assessment of boundary values related to the acoustic properties of the materials. However, the traditional techniques to measure the acoustic properties in situ suffer from geometrical constraints that prevent their application to practical situations. In contrast to existent methods, the measurement method introduced in this paper estimates the acoustic impedance of not only one but all the surfaces in a realistic interior. The method assumes that the 3D geometry of the room, the strength and location of the sound source, and a set of sound pressures measured at random locations in the interior sound field are known parameters. Moreover, the actual implementation incorporates video cameras and a 3D tracking system to facilitate the measurement process with a single microphone. Hence, the principle to estimate the acoustic impedance on the surfaces is based on the solution of an inverse linear problem arisen from the boundary element formulation applied to the interior boundaries of the room. Nevertheless, since this linear system is rank deficient, the straight solution is highly sensitive to noise and leads to wrong estimations of the acoustic impedance. On the other hand, in the proposed method, this difficulty is avoided by reformulating the inverse problem into an iterative optimization approach which takes advantage of the known geometry of the surfaces. In this paper, results of preliminary validation experiments performed in a controlled environment are presented.

1. INTRODUCTION

In situ measurement of acoustic properties of the materials has been a topic of active research and a number of approaches have been proposed since several decades ago. For example, among the most widely used techniques to measure the acoustic impedance and absorption coefficient of planar surfaces is a method based on the analysis of the sound signals recorded by a pair of microphones placed at a specific distance from the test surface and the sound source, usually a speaker – (examples of this approach [1, 2]). While this technique has been proved to be effective at the mid and high frequency ranges, it conveys geometrical constraints that prevent its application in most practical situations: *i*) the test surface should be large and flat; *ii*) the sound measurements must be performed in free space, or at least in a large space or in an anechoic room; *iii*) the microphones have to be placed close to the surface or at a suitable distance from the surface and the sound source. In efforts to overcome some of the constraints, recent methods have proposed the use of new measuring devices such as the "microflown", (a sound pressure and a particle velocity sensors encapsulated in a single package [3]), others make use of the surrounding environmental noise in order to eliminate the necessity of speakers [4]. Although these approaches lead accurate results measuring the acoustic impedance of flat surfaces, they are still unable to handle complex-shape surfaces, and in the worst case , if the test surfaces are inaccessible, they require to remove the test panels from their original place. In contrast, the approach introduced in this paper attempts to estimate the normal-incidence acoustic impedance of not only one test surface but all the surfaces contained in an enclosed space with arbitrary shapes (such as in a real room). This is possible by virtue of approaching the problem as the solution of an inverse boundary problem in which the input parameters are (1) the geometry of the interior space, (2) the location and strength of the sound source (a speaker), and (3) a set of sound pressures measured at arbitrary points in the interior field. The underlying idea is, to find the boundary values that reproduce the measured sound field by solving the system of linear equations derived from the inversion of the sound propagation model. Although this idea is not new, most approaches (the most representative is the Near-field Acoustic Holography, [5]) have been proposed for the localization and estimation of the vibration strength on the surface of sound sources. Work on the estimation of acoustic impedance of materials by inverse methods has been scarce. Nevertheless, one of the first theoretical approaches to recover the acoustic impedances of the boundaries from sound field measurements was reported in [6], in which the numerical simulations show that the method achieves meaningful results when the microphones are placed at optimum distances from the boundaries. Moreover, since the framework in [6] is based on the Finite Element (FEM) and Finite Difference (FDM) methods to model the acoustic field, the computational cost represents a serious limitation for applications in a real room. Compared with the framework in [6], the approach reported in the following paragraphs is based on the Boundary Element method (BEM) which is a more efficient numerical tool for modeling the interaction between the boundaries and the interior field they enclose. Furthermore, the linear equations derived from the BEM model in combination with the prior knowledge of the geometric segmentation of the surfaces, allow the formulation of an iterative optimization algorithm whose output is the sought acoustic impedances of the interior surfaces.

2. THEORY OF THE APPROACH

2.1. Formulation of the problem

Consider the geometry of the room depicted in Figure 1. The setup for the estimation of the acoustic impedance on the n interior surfaces $S_1, S_2, \ldots, S_n$, consists basically on a space enclosed by $S | S = S_1 \cup S_2 \cup \ldots \cup S_n$, a sound source vibrating harmonically with a normal particle velocity v_E at a frequency ω, and a microphone that measures the sound pressure p_f at different points $\mathbf{r}_f$ while freely moving in the homogeneous steady-state interior field. Fur-

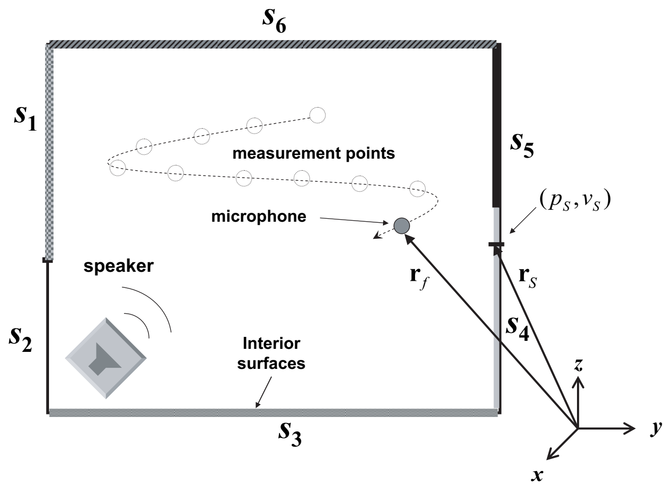


Figure 1. Geometry for the estimation of the acoustic impedance on the surfaces of a room.

thermore, the relationship between the sound pressure p_S and particle velocity v_S at any point $\mathbf{r}_S$ on S, and the sound pressure p_f at any point $\mathbf{r}_f$ in the interior ($\mathbf{r}_f \notin S$), is governed by the Helmholtz-Kirchhoff equation [7]:

$$\oint_S \left(p_S \frac{\partial \mathrm{G}(\mathbf{r}_S, \mathbf{r}_f)}{\partial \mathbf{n}} + j\omega\rho\, \mathrm{G}(\mathbf{r}_S, \mathbf{r}_f)\, v_S \right) dS + p_f = 0\,, \tag{1}$$

where the Green's function G is given by $\mathrm{G}(\mathbf{r}_S, \mathbf{r}_f) = e^{-j\mathrm{k}|\mathbf{r}_S - \mathbf{r}_f|}/4\pi|\mathbf{r}_S - \mathbf{r}_f|$, $\mathrm{k} = \omega/c$ is the wave number, $c = 340$ [m/s] and $\rho = 1.21$ [kg/m^3] are the speed of sound and the density of the air respectively, and $j = \sqrt{-1}$.

If the surface S of the 3D geometry is discretized into N elements and the sound pressure p_f is measured at M different points, it can be shown that the discrete form of the equation (1) yields two systems of linear equations, one describing the interaction between element-element on the surface (equation (2)) and the other showing the interaction between element-field point (equation (3)):

$$\begin{aligned} \mathbf{A}_S\, \mathbf{p}_S - \mathbf{B}_S\, \mathbf{v}_S &= 0\,, & (2)\\ \mathbf{A}_f\, \mathbf{p}_S - \mathbf{B}_f\, \mathbf{v}_S &= -\,\mathbf{p}_f\,. & (3) \end{aligned}$$

For isoparametric elements, the components of the matrices $\mathbf{A}$'s and $\mathbf{B}$'s are calculated with the formulae

$$a_{i,k} = \oint_{s_k} \frac{\partial \mathrm{G}(\mathbf{r}_S, \mathbf{r}_f)}{\partial \mathbf{n}}\, ds \quad , \quad b_{i,k} = -\,j\omega\rho \oint_{s_k} \mathrm{G}(\mathbf{r}_S, \mathbf{r}_f)\, ds\,, \tag{4}$$

in which $i = 1, 2, \ldots, M$, $k = 1, 2, \ldots, N$, and s is the surface of each discrete element. In addition, since the vibration velocity of the sound source is given, the coefficients of the corresponding columns can be taken from $\mathbf{B}_f$ and the terms of equation (3) are separated into unknown and known parameters as follows:

$$\mathbf{A}_f \mathbf{p}_S - \widetilde{\mathbf{B}}_f \widetilde{\mathbf{v}}_S = \widehat{\mathbf{B}}_f \widehat{\mathbf{v}}_S - \mathbf{p}_f\,, \tag{5}$$

denoting by $\widetilde{\mathbf{v}}_S$ and $\widehat{\mathbf{v}}_S$ the unknown and known surface velocities respectively.

Solution of the linear system (5) using the known information **A**'s and **B**'s (from the geometry), $\widehat{\mathbf{v}}_S$ (the sound source) and $\mathbf{p}_f$ (the set of measured sound pressures), leads us to know the acoustic impedance z_k at each k-th discrete element, which is given by

$$z_k = \frac{p_{s_k}}{v_{s_k}} \tag{6}$$

2.2. iterative estimation of the acoustic impedances

Previous studies on similar acoustical inverse problems have shown that the inversion of the propagation model (1) results in an ill-conditioned and rank deficient linear system of equations which is sensitive to perturbations (noise), and therefore, special techniques known as *regularization* have to be employed, (for example as in [8, 9]). However, it is possible to find the acoustic impedances z_k in a more straightforward way by rewriting equation (5) and stating the problem as an iterative optimization.

Let us note first that, prior knowledge of the geometry of the surfaces $S_1, S_2, \ldots, S_n$ allows us to group the discrete elements within its corresponding surface. Assuming that the surfaces have been segmented by homogeneity[1], and considering only the local impedance effect (local reaction) at each element on the surface, the following approximations hold:

$$\frac{p_{S_i,1}}{v_{S_i,1}} \approx \frac{p_{S_i,2}}{v_{S_i,2}} \approx ... \approx \frac{p_{S_i,m_i}}{v_{S_i,m_i}} \ , \tag{7}$$

or

$$z_{S_i,1} \approx z_{S_i,2} \approx \ldots \approx z_{S_i,m_i} = Z_i \ , \tag{8}$$

where m_i indicates the number of discrete elements that belong to the i-th surface. Furthermore, noting that $\mathbf{p}_S$ of equation (5) can be substitute by their impedance effect, and grouping the columns of A_f as dictated by (8), the linear system (5) can be rewritten as:

$$\begin{pmatrix} \sum_{k=1}^{m_1} a_{f,(1,k)}\ \tilde{v}_{S,k} & \sum_{k=m_1+1}^{m_2} a_{f,(1,k)}\ \tilde{v}_{S,k} & \cdots & \sum_{k=m_{n-1}+1}^{m_n} a_{f,(1,k)}\ \tilde{v}_{S,k} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{k=1}^{m_1} a_{f,(M,k)}\ \tilde{v}_{S,k} & \sum_{k=m_1+1}^{m_2} a_{f,(M,k)}\ \tilde{v}_{S,k} & \cdots & \sum_{k=m_{n-1}+1}^{m_n} a_{f,(M,k)}\ \tilde{v}_{S,k} \end{pmatrix} \mathbf{z} \ - \ \tilde{\mathbf{B}}_f \tilde{\mathbf{v}}_S = \hat{\mathbf{p}}_f \ ,$$

where:

$$\begin{aligned} \mathbf{z} &= \{Z_1, Z_2, \ldots, Z_n\}^{\mathrm{T}} \ , \\ \hat{\mathbf{p}}_f &= \hat{\mathbf{B}}_f \hat{\mathbf{v}}_S - \mathbf{p}_f \ . \end{aligned}$$

Written in a compact form:

$$\langle \mathbf{A}_f \cdot \tilde{\mathbf{v}}_S \rangle \, \mathbf{z} - \tilde{\mathbf{B}}_f \tilde{\mathbf{v}}_S = \hat{\mathbf{p}}_f \ , \tag{9}$$

From equation (9), the acoustic impedance values **z** are estimated iteratively according

[1] Besides the geometry of the room, the appearance such as texture and color can be used to infer homogeneous regions.

1: Using $\mathbf{z}^{(0)}$, solve for $\tilde{\mathbf{v}}_S^{(\ell)}$ from

$$(\mathbf{C}_S - \tilde{\mathbf{B}}_S)\tilde{\mathbf{v}}_S = \hat{\mathbf{B}}_S\hat{\mathbf{v}}_S \tag{10}$$

2: **while** condition (13) is **false do**

3: Update **z** by

$$\mathbf{z}^{(\ell+1)} = \arg\min \|\langle \mathbf{A}_f \cdot \tilde{\mathbf{v}}_S^{(\ell)}\rangle\, \mathbf{z} - \tilde{\mathbf{B}}_f\tilde{\mathbf{v}}_S^{(\ell)} - \hat{\mathbf{p}}_f\| \quad \text{s.t. } \mathrm{Z}_{\min} \leq \mathbf{z} \leq \mathrm{Z}_{\max} \tag{11}$$

4: Update $\tilde{\mathbf{v}}_S^{(\ell+1)}$ using $\mathbf{z}^{(\ell+1)}$ in Eq. (10)

5: Compute $\mathbf{p}_g$ by

$$\mathbf{p}_g = \hat{\mathbf{B}}_f\hat{\mathbf{v}}_S - (\mathbf{C}_f - \tilde{\mathbf{B}}_f)\tilde{\mathbf{v}}_S^{(\ell+1)} \tag{12}$$

6: Evaluate

$$h(\mathbf{p}_g, \mathbf{p}_f) \leq \alpha \tag{13}$$

7: $\ell \leftarrow \ell + 1$

8: **end while**

Algorithm 1. Iterative estimation of **z**

to the steps shown in Algorithm 1. The iterative process starts with the initial guess $\mathbf{z}^{(0)} = Z_1^{(0)}, Z_2^{(0)}, \ldots, Z_n^{(0)}$. If the evaluation of the objective function $h(\mathbf{p}_g, \mathbf{p}_f)$ does not satisfy the criterion α, the vector **z** is updated at each iteration as indicated in step 3 of Algorithm 1. Here $h(\mathbf{p}_g, \mathbf{p}_f)$ is defined as

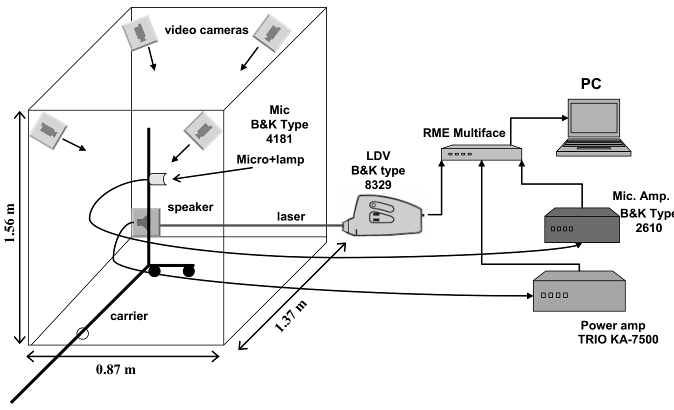
$$h(\mathbf{p}_g, \mathbf{p}_f) = \frac{\|\mathbf{p}_g - \mathbf{p}_f\|^2}{\|\mathbf{p}_f\|^2} + \frac{1}{M-1}\frac{\|\mathbf{p}_G - \mathbf{p}_F\|^2}{\|\mathbf{p}_F\|^2}\,, \tag{14}$$

where $\mathbf{p}_g$ is a set of sound pressures predicted at the same points of $\mathbf{p}_f$ using $\mathbf{z}^{(l+1)}$ through steps 4 and 5. The vectors $\mathbf{p}_G$ and $\mathbf{p}_F$ are respectively the sound pressures $\mathbf{p}_g$ and $\mathbf{p}_f$ taken in dB's. In (10) and (12), the entries of the matrices **C**'s are computed as $c_{i,k} = z_{S,k} \cdot a_{i,k}$. At step 3, the update of $\mathbf{z}^{(l+1)}$ requires the solution of the optimization problem given by (11) which is solved by a global optimization algorithm based on Sequential Quadratic Programming (SQP). The routine that implements this solver is available as a function in the comercial software Matlab. In addition, the SQP solver is constrained to seek a solution in the space bounded by $\mathrm{Z}_{\min}$ and $\mathrm{Z}_{\max}$. When the acoustic impedances are written in terms of absorption coefficients, positive values of the latter are expected, therefore, the bound constraints become $0 < \mathrm{Re}\{\mathbf{z}\} < \mathrm{Z}_{\max}$ and $\mathrm{Z}_{\min} < \mathrm{Im}\{\mathbf{z}\} < \mathrm{Z}_{\max}$, where $\mathrm{Z}_{\min}$ can be specified as $\mathrm{Z}_{\min} = -\mathrm{Z}_{\max}$, and $\mathrm{Z}_{\max}$ as the acoustic impedance of a hard wall (e.g. concrete).

3. EXPERIMENTS IN A CONTROLLED ENVIRONMENT

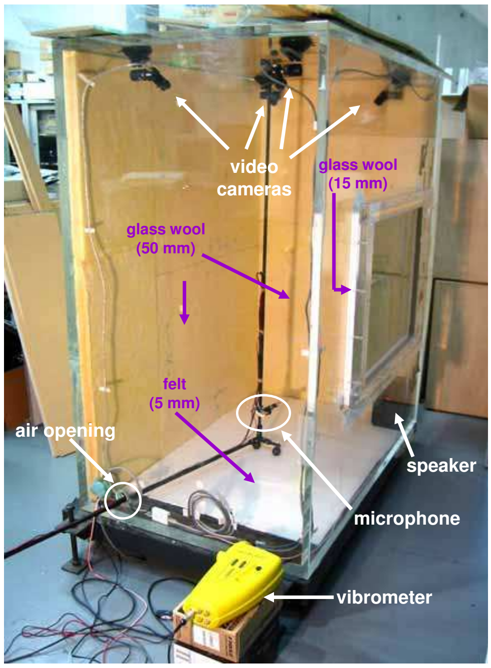
Preliminary validation experiments have been conducted in a controlled environment produced in the interior of a reverberation chamber whose rigid walls are made of 3 mm-thick acrylic, and with dimensions shown in the experimental setup of Figure 2a.

The experiment consists on attempting to estimate the acoustic impedance on the surface of the absorbent materials placed in the interior of the chamber as follows: two lateral walls covered with glass wool (50 mm–32 Kg/m^3), felt (5 mm–96 Kg/m^3) on the floor, and a lateral window of glass wool (15 mm–96 Kg/m^3). All materials are rigid-backed by the walls of the



a)

Figure 2. a) Diagram with the equipment used for the experiments. b) Setup in the actual experimental chamber.



b)

chamber, (see Figure 2b).

Observe that as the speaker emits a tone of frequency f, its vibration amplitude $|v_E|$ is measured by the Vibrometer, and the sound pressure can be measured continuously through the microphone while moving freely. Moreover, in order to acquire the coordinates of the measured points, 3D-tracking on the microphone is performed in real time using the four overhead video cameras. In this way, as many as desired measurements of sound pressure at arbitrary points in the interior can be acquired. Nonetheless, from the Doppler's relation, the maximum speed μ (in m/s) at which the microphone can be displaced is constrained by

$$\mu \leq c\left(\frac{f+|\beta|}{f}-1\right) , \tag{15}$$

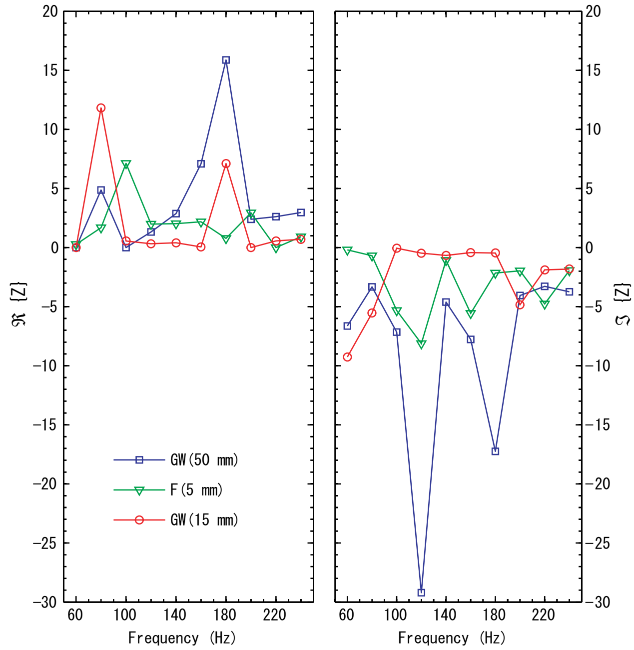
here c is the velocity of sound in air, and β is the tolerated frequency deviation (in Hz) of the signal observed at the microphone. In the experiments, the test frequencies f were chosen from 60 Hz to 240 Hz with steps of 20 Hz and, for each frequency, the sound pressure was measured at $M = 2N$ arbitrary points.

4. RESULTS

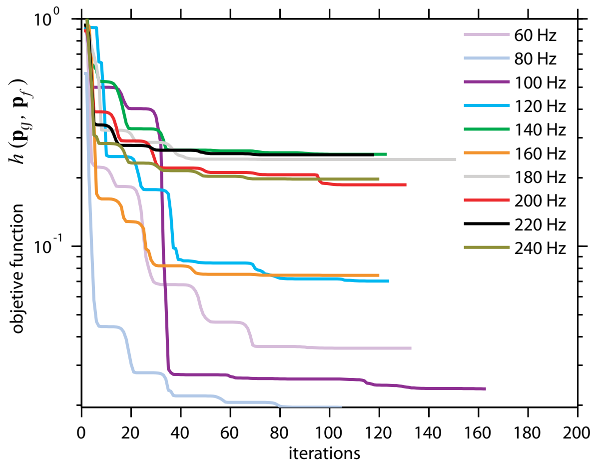
In the data processing stage, the geometry of the chamber was modeled with a 3D mesh of 1446 isoparametric-triangular elements whose maximum edge size is 0.12 m, allowing for BEM analyses up to 470 Hz in agreement with the rule of six elements per wavelength. Moreover, the surfaces of the absorbents were modeled accordingly to their original geometry. The walls of the chamber without any absorbent were assumed to be rigid boundaries. To initialize the iterative optimization, $\mathbf{z}^{(0)} = \mathbf{1}$ and $\alpha = 0.01$ were specified.

The Algorithm 1 converged to the normalized acoustic impedances shown in Figure 3a after a number of iterations, as can be seen in Figure 3b. Let us note that the iterations were

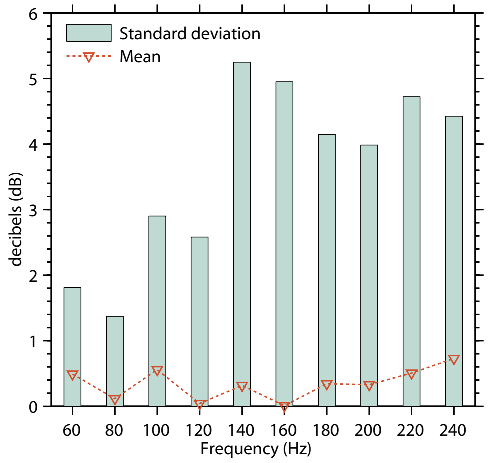
Figure 3. Experimental results. a) Normalized acoustic impedances ($\mathbf{z}/\rho c$) of the glass wool 50 mm [GW(50 mm)], the felt 5 mm [F(5 mm)] and glass wool 15 mm [GW(15 mm)]. b) Convergence history of the Algorithm 1. c) Standard deviation and mean of the error $|\mathbf{p}_G - \mathbf{p}_F|$.



a)



b)



c)

stopped by a minimum-improvement criterion, since convergence to the specified $\alpha = 0.01$ was not achieved in any case. Nevertheless, as Figure 3c suggests, the estimated acoustic impedances of Figure 3a are able to reproduce the actual sound field with a maximum average error of 0.7 dB's (for 240 Hz), and in the worst case, with a standard deviation of 5.2 dB's (for 140 Hz). Thus, because the validation of the estimated impedance values turns out to be difficult because of the lack of ground-truth data[2], the rates of Figure 3c can give margins of the expected error when performing numerical simulations using the values of Figure 3a within the specified frequencies.

[2]By the time of preparation of this paper, there is still no other practical method to estimate the acoustic impedance of the interior surfaces under similar experimental conditions.

5. CONCLUSIONS

A method for the estimation of the acoustic impedance on the arbitrary-shape surfaces of interiors was introduced. In contrast to previous approaches, the method in this paper has been tested experimentally and, although its validation is ongoing, the preliminary results suggest that indeed, using an inverse BEM-based approach, it is possible to estimate the normal acoustic impedance of the surfaces from a set of sound pressures measured at arbitrary points in the interior steady-state field. Moreover, the incorporation of video cameras effectively facilitates the measurement process. As a result, the implemented system overcomes most of the constraints that traditional methods suffer. On the other hand, a condition to keep in mind is that the surfaces in the room should be fundamentally local reactive. Note however, that the method can be generalized to deal with extended-reaction surfaces by including a suitable model (e.g. FEM) for those surfaces in combination with the general BEM model. Furthermore to note is that improvements on the computational burden are also possible. Since the Algorithm 1 does not require the storage of the BEM matrices for the evaluation of the objective function $h(\mathbf{p}_g, \mathbf{p}_f)$, highly efficient frameworks such as Fast Multipoles (e.g. FMBEM) can be used instead of the BEM-based framework adopted in this work.

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