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An inverse method for in-situ estimation of acoustic surface impedance targeting inverse sound rendering in rooms

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ABSTRACT

We propose two methods based on the inverse boundary element method (IBEM) for the estimation of acoustic impedances of the interior surfaces of a room. The algorithms take as input the geometrical model of the space, a set of sound field pressures measured at random points and the vibration strength of the sound source. A first approach is a constrained least-squares method that attempts to find simultaneously surface sound pressures and particle velocities, parameters that define the acoustic impedance. This is in contrast with previous work on similar acoustic inverse problems that solve for one parameter (typically the strength of the vibration velocity) and use regularization techniques to reduce the effects of the ill-conditioning and obtain smooth solutions. Our second approach is an iterative optimization process that estimates directly the sought impedances assuming that the interior surfaces have homogeneous impedance values. This assumption allows in addition a significant reduction of the dimensionality of the optimization problem. A numerical example of the performance of both methods is presented. Preliminary experiments in a reverberation chamber are also shown.

1 INTRODUCTION

Measurement of acoustic properties of the materials is important in the prediction of sound fields with numerical techniques such as finite element methods (FEM) and boundary element methods (BEM) where boundary values should be specified for the solution of vibro-acoustic parameters. These boundary values can be measured directly from sample materials in a laboratory, however there are situations in which the measurements must be performed in the actual place of the materials. One of the techniques to approach this problem consists on taking samples of sound pressures in the near-field of the surface under analysis and using this data to solve the inverse of a transfer function that relates the measured data with the parameters sought. This kind of acoustic inverse problem is well known in the exterior radiation reconstruction where the BEM-based near-field acoustic holography (BEM-based NAH) technique [1] is used to reconstruct the sound strength on the surface of a vibrating object. In the radiation problem the BEM equations are solved for one of the surface parameters (typically the particle velocity v_s , since the goal is the sound strength reconstruction), and the other one is calculated consequently. Furthermore, due to the problem of ill-conditioning, methods such as truncated-SVD and Tikhonov regularization have been widely employed to reduce the amplification of the noise propagated to the found solution, (e.g. [3],[4]). We introduce two approaches that attempt to

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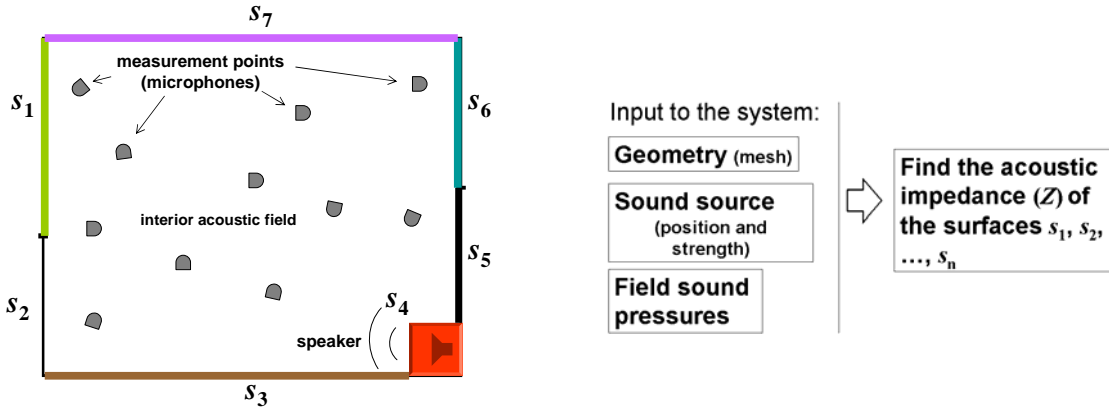


Figure 1. The inverse problem of the estimation of acoustic impedance at the surfaces of a room.

solve the inverse BEM (IBEM) interior problem of finding the acoustic normal impedance of the surfaces for inverse sound rendering [7]. The proposed methods differ from existent techniques in that they attempt to estimate the acoustic impedance of all the surfaces in a room simultaneously by recording samples of sound while freely moving one microphone around the interior space, in contrast to other methods that measure the acoustic impedance of one surface requiring the use of two or more microphones placed at locations close to the surface under analysis.

2 IBEM FORMULATION FOR THE IN-SITU ESTIMATION OF ACOUSTIC IMPEDANCE

2.1 Definition of the inverse problem

As depicted in Figure 1, given the 3D geometric model of a room, the position and vibration strength of a time-harmonic sound source (e.g. a sinusoidal tone of frequency ω), and a set of M field sound pressures p_f measured at known random positions in the room, our task is to find the acoustic impedances Z_S of the interior surfaces S_1, S_2, \dots, S_n , where at a point x on a surface the Z_S is defined by the ratio of sound pressure p_S and particle velocity v_S as

$$Z_{S_x} = \frac{p_{S_x}}{v_{S_x}} \quad (1)$$

On the other hand, discretization of the 3D model into N elements yields the discrete form of the Kirchhoff-Helmholtz equation, and employing the acoustic BEM formulation as in [2][7], we arrive to the following basic equations that relate the surface sound pressures \mathbf{p}_S and the particle velocities \mathbf{v}_S with the sound pressures \mathbf{p}_f in the interior field:

$$\mathbf{A}_S \mathbf{p}_S + \mathbf{B}_S \mathbf{v}_S = 0 \quad (2)$$

$$\mathbf{p}_f + \mathbf{A}_f \mathbf{p}_S + \mathbf{B}_f \mathbf{v}_S = 0 \quad (3)$$

where $\mathbf{A}_S, \mathbf{B}_S$, are $N \times N$ surface-surface influence matrices, and $\mathbf{A}_f, \mathbf{B}_f$, are $M \times N$ surface-field influence matrices. \mathbf{p}_S and \mathbf{v}_S are $N \times 1$ vectors and \mathbf{p}_f is a $M \times 1$ vector. The influence matrices \mathbf{A} 's and \mathbf{B} 's are purely dependent on the geometry of the model and the position of the measurement points, thus they can be computed in advance.

According to Eq. (1), in order to estimate the surface acoustic impedances we have to solve the systems of Eqs. (2)-(3) for the unknown parameters \mathbf{p}_S and \mathbf{v}_S . Typically one may proceed to eliminate one variable by substitution and solve the resulting reduced system. However, here we will attempt to find the solution with two approaches, both involving a nonlinear optimization process: the first estimates \mathbf{p}_S and \mathbf{v}_S simultaneously allowing us to define bounds to the sought solution; the second tries to find Z_S iteratively exploiting knowledge of the surface segmentation in the 3D model and most importantly avoiding the inversion of the ill-conditioned matrix that is inherent in this kind of inverse acoustic problems.

2.2 Least-squares estimation of \mathbf{p}_S and \mathbf{v}_S

Since the vibration velocity $\hat{\mathbf{v}}_S$ at the sound source and the field pressures are known, we can rewrite Eqs. (2) and (3) as

$$\mathbf{A}_S \mathbf{p}_S + \tilde{\mathbf{B}}_S \tilde{\mathbf{v}}_S = -\hat{\mathbf{B}}_S \hat{\mathbf{v}}_S \quad (4)$$

$$\mathbf{A}_f \mathbf{p}_S + \tilde{\mathbf{B}}_f \tilde{\mathbf{v}}_S = -\hat{\mathbf{B}}_f \hat{\mathbf{v}}_S - \mathbf{p}_f \quad (5)$$

having unknowns \mathbf{p}_S and $\tilde{\mathbf{v}}_S$, and known variables \mathbf{p}_f and $\hat{\mathbf{v}}_S$ in the left and right side respectively.

The unknowns \mathbf{p}_S and $\tilde{\mathbf{v}}_S$ of Eqs. (4) and (5) can be grouped in one vector and the augmented system of equations is therefore expressed as

$$\begin{pmatrix} \mathbf{A}_S & \tilde{\mathbf{B}}_S \\ \mathbf{A}_f & \tilde{\mathbf{B}}_f \end{pmatrix} \begin{pmatrix} \mathbf{p}_S \\ \tilde{\mathbf{v}}_S \end{pmatrix} = - \begin{pmatrix} \hat{\mathbf{B}}_S \hat{\mathbf{v}}_S \\ \hat{\mathbf{B}}_f \hat{\mathbf{v}}_S + \mathbf{p}_f \end{pmatrix}, \quad (6)$$

or compactly:

$$\mathbf{D} \mathbf{x}_S = \mathbf{d}. \quad (7)$$

Furthermore, assuming that the acoustic impedance of all the interior surfaces is bigger than that of the propagation media (we will consider that room is filled with air, thus $Z_0 = \rho c \approx 415$ rayls) and imposing a maximum impedance value (e.g. impedance of concrete, steel, etc.), we can define lower and upper bounds in which the solution should be found. Therefore the least-squares minimization problem is stated as follows:

$$\begin{aligned} & \min_{\mathbf{x}_S} \|\mathbf{D} \mathbf{x}_S - \mathbf{d}\|^2 & (8) \\ & \text{s.t. } Z_0 \leq z_i(x_{S,i}) \leq Z_{\max} \\ & \text{with } z_i(x_{S,i}) = \frac{p_{S,i}}{v_{S,i}}, \\ & \text{and } i = 1, 2, \dots, N, \end{aligned}$$

It is now clear that the imposition of bounds implicitly transforms the linear system of equations given by (4) and (5) into a nonlinear minimization problem that requires nonlinear

iterative solvers for its solution. We note also that at least $M = N$ field pressure measurements are required by this minimization problem.

2.3 Iterative estimation of the acoustic impedance

Although the least-squares approach presented above gives approximated solutions, it is still unable to exploit extra information about the surfaces, thus returning solutions with large levels of variance due to the ill-conditioning of the matrix \mathbf{D} . In order to eliminate these variances we start from the assumption that the acoustic impedance is homogeneous in each type of surface of the 3D model. Then, replacing the surface pressures p_S of Eq. (3) by Z_S from Eq. (1) we can express Eq. (3) as follows:

$$\mathbf{A}_f \langle \mathbf{v}_S \cdot \mathbf{z}_S \rangle = -\mathbf{B}_f \mathbf{v}_S - \mathbf{p}_f \quad (9)$$

where $\langle \cdot \rangle$ means element-wise product.

As described in the definition of the problem, there are n different surfaces in the room, and the above assumption implies that for each l -th surface

$$z_{S, m_{l-1}+1} = z_{S, m_{l-1}+2} = \dots = z_{S, m_l} = Z_l \quad , \quad \text{with } l = 1, 2, \dots, n \quad (10)$$

here, $z_{S, m}$ are the surface impedance values at each element of the mesh, and Z_l is the homogeneous impedance of the surface S_l . Substituting the $z_{S, m}$'s by their corresponding Z_l 's we can reduce the system of Eqs. (9) to a system with n unknown parameters as follows:

$$\begin{pmatrix} \sum_{j=1}^{m_1} a_{f1,j} v_{S,j} & \sum_{j=m_1+1}^{m_2} a_{f1,j} v_{S,j} & \cdots & \sum_{j=m_{n-1}+1}^{m_n} a_{f1,j} v_{S,j} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{j=1}^{m_1} a_{fM,j} v_{S,j} & \sum_{j=m_1+1}^{m_2} a_{fM,j} v_{S,j} & \cdots & \sum_{j=m_{n-1}+1}^{m_n} a_{fM,j} v_{S,j} \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_n \end{pmatrix} = -\mathbf{B}_f \mathbf{v}_S - \mathbf{p}_f \quad (11)$$

or

$$\hat{\mathbf{A}}_f \mathbf{z} = -\mathbf{B}_f \mathbf{v}_S - \mathbf{p}_f \quad (12)$$

where the a_f 's are the elements of the matrix \mathbf{A}_f .

Prescribing an initial \mathbf{z}_{init} we start an iterative process by substituting this initial vector into the system of Eqs. (2)-(3) and then generating a set of M field pressures \mathbf{p}_g at the same positions of \mathbf{p}_f , hence the following cost function is evaluated at each iteration:

$$\alpha = \frac{\| \mathbf{p}_g - \mathbf{p}_f \|^2}{\| \mathbf{p}_f \|^2} \leq \text{tol} \quad , \quad (13)$$

where $\| \cdot \|$ indicates the L_2 norm, and tol is a termination threshold. If Eq. (13) is not satisfied, Eq. (12) is updated with the predicted \mathbf{v}_S and solved for a new \mathbf{z} . Note that the dimensionality of the optimization problem has been reduced from $2N$ (for the case of the previous least-squares

approach) to n (where typically $n \ll N$). Moreover, we may impose again the bounds used in the least-square solution of the previous section to give more information of the desired solution. Thus the iteration process takes the form:

$$\begin{aligned} \mathbf{z}^{(k+1)} = \min_{\mathbf{z}} & \left\| \hat{\mathbf{A}}_f^{(k)} \mathbf{z} + \mathbf{B}_f \mathbf{v}_s^{(k)} + \mathbf{p}_f \right\| \\ \text{s.t.} & \quad Z_0 \leq \mathbf{z} \leq Z_{\max} \end{aligned} \quad (14)$$

k being here the iteration index. The actual implementation of the algorithm was done in Matlab.

3 NUMERICAL EXAMPLE

3.1 Numerical simulation: Reverberation chamber

The performance of the described algorithms is investigated first through a numerical example, and in further sections a real experimental case will be presented.

For the simulation we use the graphic model of an experimental reverberation chamber. This chamber consists of a 28mm-thick acrylic box of dimensions 1.37x1.56x0.87m. Its 3D model is produced and meshed into a 314 isoparametric triangular elements with a maximum size of 0.22 m, allowing a frequency analysis of up to 260 Hz with $\lambda/6$ rule. Figure 2 depicts the model of the chamber. In order to generate 1000 field pressures at randomly distributed points, normalized real valued impedances are manually assigned to the interior surfaces as follows: 1000 for the walls, 2000 for the sides of the speaker and 1 for the lateral opening of the chamber (note that these values are normalized to Z_0). It is assumed that only the circular area of the speaker is the vibrating sound source. Under these conditions the set of field pressures are generated using BEM for a frequency of 250 Hz.

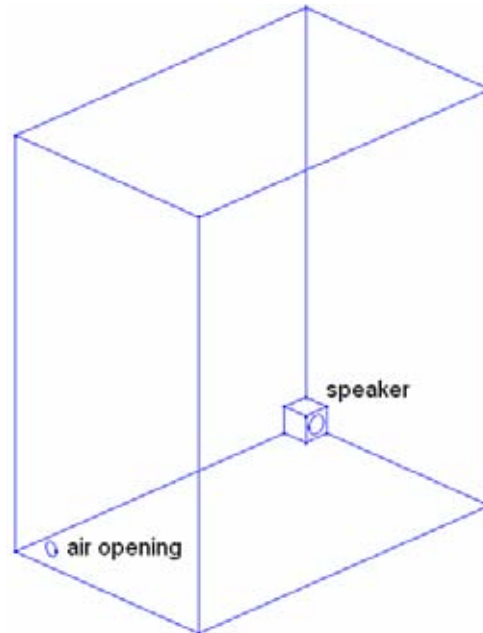


Figure 2. Geometric model of the reverberation chamber.

Because noise from different sources is introduced in practical measurements, we simulated this effect by adding artificial noise to the computed pressures and use them now for the estimation of the impedances using the two approaches described. The analysis is performed for three cases of noise level. These are: a) Noiseless, b) SNR = 50 dB and c) SNR = 30 dB.

For the case of the iterative algorithm, four types of surfaces are specified: the air opening, the walls of the chamber, the sides of the speaker and the circular vibrating surface of the speaker. The latter is considered as a known variable and therefore the number of unknown impedance values to be determined in the iterative process is $n = 3$. An initial value of $\mathbf{z} = \mathbf{Z}_0$ is chosen to start the iterative optimization.

3.2 Results of the numerical example

In Figure 3 we compare the results obtained from this simulation example. It is observed that the constrained least-squares method gives approximated solutions when the data is not severely contaminated with noise, however the element-wise values of these solutions tend to vary greatly as the noise level grows. The poor smoothness of the solutions of this method is due to the amplification of noise due to the ill-conditioning that is inherent to this kind of problems. The mechanism of amplification of noise in the back reconstruction has been studied extensively in the literature using SVD analysis, e.g. [8][9]. On the other hand, it was found that the least-square approach is more efficient than the iterative one in the time required to find the solution. It has been shown in [7] that for cases when the level of noise is almost null, standard techniques such as Gaussian elimination and Least-squares with QR-decomposition, which are highly computationally efficient solvers, can give comparable results to those using regularization techniques such as Tikhonov treatment [5].

The performance of the iterative method was found to be more robust to the noise contained in the data, achieving fairly close solutions even when the SNR = 50 dB. Another advantage is the suppression of element-wise high variance of the estimated impedance values, since only one value of impedance is sought for each type of surface. This brings the consequent benefit of reducing considerably the dimensionality of the inverse problem. A draw back of this method is its poor convergence, taking in all of the cases studied here between 240 and 500 iterations until a solution is found. In the current implementation of this algorithm no information of the gradient of the cost function, nor a search method is employed to accelerate the convergence, nevertheless in practice it was found that the method converges. The convergence of this approach is also sensitive to the starting point, i.e. the initial value specified. Thus the selection of an appropriate initial vector is another limitation of this method.

4 PRELIMINARY EXPERIMENT

4.1 Experimental setup

An attempt to validate practically the effectiveness of the introduced algorithms has been done by performing measurements of field pressures in the interior of a reverberation chamber. The characteristics of the chamber have been given in the previous section.

A 10 cm-cubic speaker has been used as the sound source excited by a sinusoidal signal, and its vibration amplitude was measured with a Laser Doppler Anemometer (LDA). In order to take large sets of field pressures, continuous sampling was performed while moving the microphone freely in the interior of the box. 3D stereo tracking with an array of

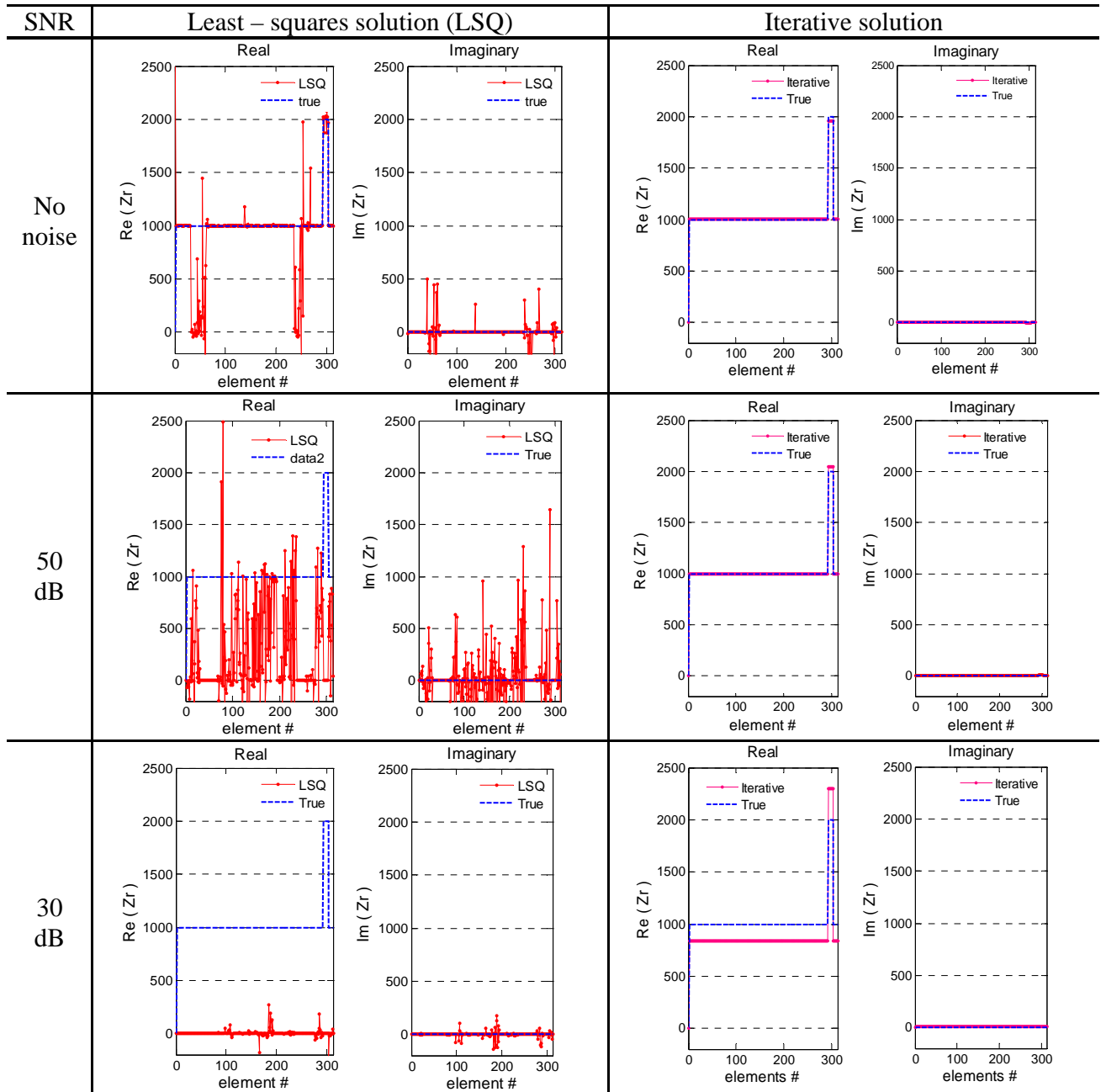


Figure 3. Comparison chart of the estimation of surface impedance by constrained least-squares (LSQ) and by iterative optimization (Iterative). The impedance values are relative to $Z_0 = \rho c$, and displayed element-wise in their real and imaginary part.

small cameras was employed to acquire the position of the microphone in real-time. Figure 4 shows the installation of the devices used for the experiments.

Once the speaker is outputting a tone, the chamber is closed and one can move the microphone from the outside using a carrier. An omnidirectional microphone was used and attached to the carrier at different heights. It was then moved randomly in the horizontal plane.

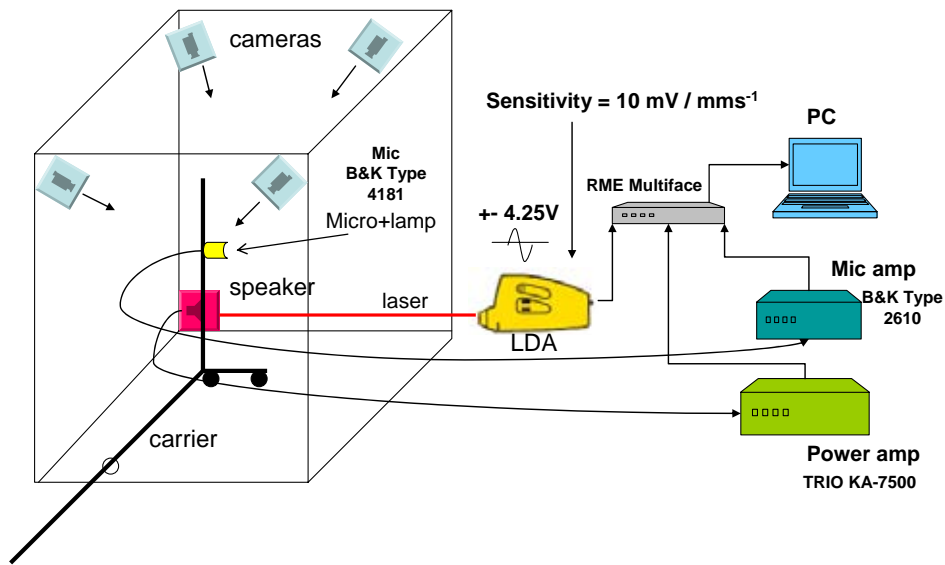


Figure 4. Setup of the devices employed for the experiments.

Care was taken when moving this microphone so as to minimize the Doppler effect in the collected data.

The same procedure was repeated for tone frequencies of 125 Hz, 250 Hz, 500 Hz and 1 kHz and a mesh with a maximum element size of 0.057 m is used. Because of the continuous sampling facility, sets of 5000 field pressures were taken at each frequency. In an off-line process, points with certain level of 3D tracking error are filtered, and some more with spurious pressure phase values are discarded. Figure 5 shows an example of the 3D positions acquired by the stereo tracking system.

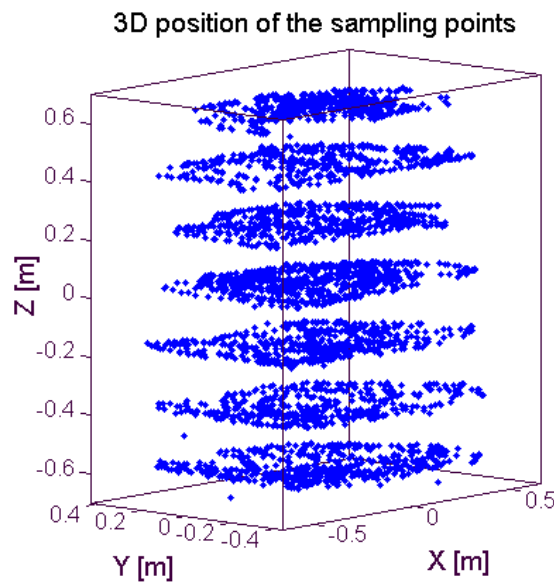


Figure 5. Example of the 3D points acquired by the stereo tracking system in the interior of the chamber.

With the measured field pressures, the inverse estimation of the acoustic impedance is performed. In this experiment an empty acrylic box has been considered and thus, similarly to the simulations presented, the number of impedance values sought is limited to $n = 3$ (i.e. the air opening, the walls and the rigid sides of the speaker).

4.2 Results of the experiment

As demonstrated by the results of the simulations in Figure 3, in the presence of noise the least-squares approach failed to find a reasonable impedance value from the experimental data, and thus this result is omitted. The iterative method, in contrast, was able to arrive to a solution. The impedances values obtained by the iterative approach are shown in Figure 6.

In practice, it has been difficult to measure the actual impedance value of the walls in the interior of the reverberation box by employing other techniques different from the ones introduced in this work, and therefore we lack of reference data for a proper comparison and validation of our approach by the moment. Nevertheless, as expected, the estimated impedance values display a large absolute value for the case of the walls and the sides of the speaker since these surfaces are basically rigid boundaries. Accordingly, also the impedance at the air opening area is near the value of Z_0 .

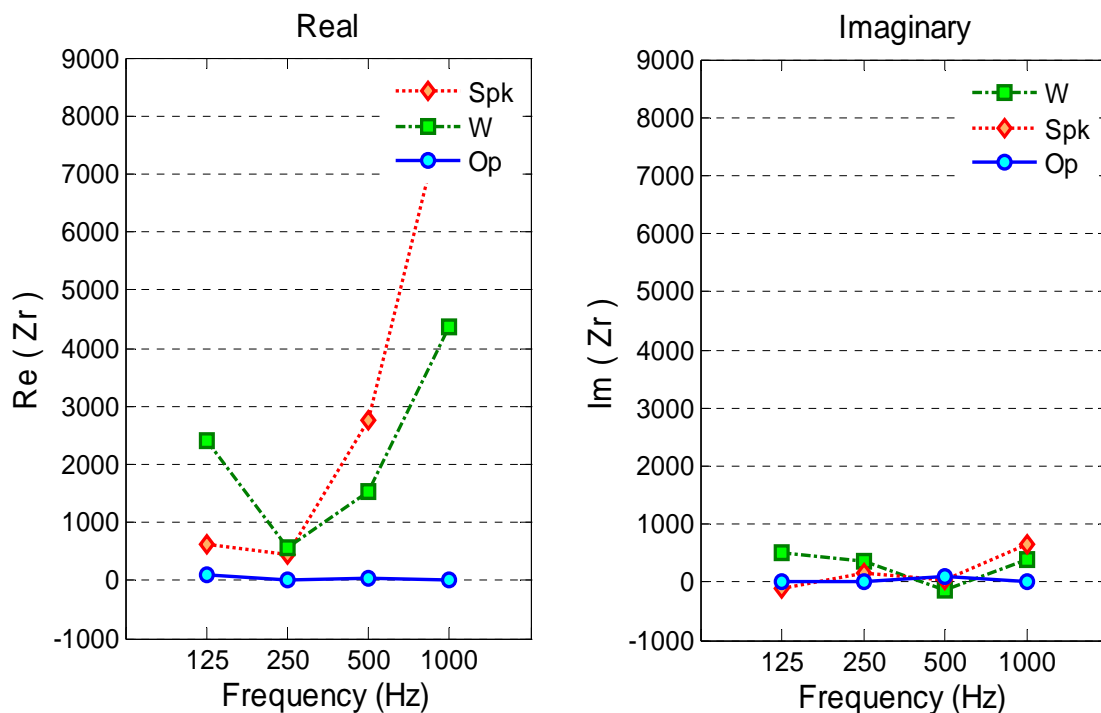


Figure 6. Experimental impedance values estimated by the iterative approach. Surface types: **Spk** – rigid sides of the speaker, **W** – rigid walls of the chamber, **Op** – air opening in a lateral side of the chamber.

5 CONCLUSIONS

Two approaches for the in-situ estimation of the acoustic impedances of the surfaces were studied. Although both are based on the inversion of the BEM the solution of the system of equations differs fundamentally on the parameters to be estimated. While the constrained least-squares solves for the surface pressure and the particle velocity at each element simultaneously, the iterative approach solves directly for the sought impedance values, reducing considerably the dimensionality of the problem from $2N$ to m , where N is the number of elements in the mesh of the geometric model and m is the number of interior surfaces of distinct materials. Moreover, the inversion of the near-singular matrix in the system of equations is avoided. The ill-conditioning, however, is still present and affects the performance of the iterative optimization process. This is specially observed in the slow convergence of the algorithm due to the existence of many solutions that minimize the cost function.

The final challenge is to perform experiments in real rooms where there are a number of surfaces with complex geometries. In principle, the BEM is able to handle irregular geometries provided that a suitable mesh is used. On the other hand, the effects of an increase of the number of surfaces over the iterative solutions will be also investigated.

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